(3 Hours) **[Total Marks: 80]**

Note: All Questions are compulsory
Use of simple calculator is allowed
Figure at right indicate maximum marks

Q.1 **(a)** **Attempt any 7 [2 marks each]** **[14]**

(i) N^{th} derivative of y = $\frac{1}{9x+2}$ is

(a) $\frac{(-1)^{n-1}(n-1)!9^n}{(9x+2)^n}$ (b) $\frac{(-1)^n(n)!9^n}{(9x+2)^{n+1}}$ (c) $\frac{(-1)^{n-1}(n-1)!9^n}{(9x+2)^{n+1}}$ (d) $\frac{(-1)^n(n)!9^n}{(9x+2)^n}$

(ii) $\int_{-2}^{2} x^5$ dx is;

(a) $\frac{16}{3}$ (b) $\frac{8}{3}$ (c) 0 (a) $\frac{3}{16}$

(iii) The differential equation for the function y = mx + c is

a) $x\frac{dy}{dx}$ - y = 0 (b) $\frac{d^2y}{dx^2} = m$ (c) $\frac{d^2y}{dx^2} = 0$ (d) $2x\frac{dy}{dx}$ + m= 0

(iv) If A=$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ k & 2 & 4 \end{bmatrix}$ is a singular matrix, then the value of x is:

(a) 1 (b) 2 (c)4 (d) 6

(v) A group of 50 observations has A.M =61 and S.D=8. Another group of 100 observations has A.M=70 and S.D=9, then the A.M for combined group of 150 observations.

(a) 67 (b)77 (c) 87 (d) 97

(vi) For a binomial distribution, mean=4 and variance=2.4, then the value of parameters n and p are

(a) 8 and 0.5 (b) 10 and 0.6 (c) 10 and 0.4 (d) None of these

(vii) The table value for a Normal distribution, P[Z ≥ 2.1]=0.0179 then P[Z ≤ 2.1]=

(a) 0.4821 (b) 0.9821 (c) 0.0179 (d) None of these

(viii) $\int \frac{cosx}{5+sinx}$ dx is

(a) log $|5 + sinx|$ +c (c) log x +c

(b) log $|sinx|$ +c (d) log $|cosx|$ +c

(ix) The solution of the differential equation x dx + y dy = 0 is:

(a) $x^2 + y^2 = c$ (b) $x^2 - y^2 = c$ (c) x + y = c (d) x - y = c

(b) **Attempt any 1:** **[1]**

(x) If A = $\begin{bmatrix} 3 & x & y \\ 1 & 2 & z \\ -2 & 2 & 4 \end{bmatrix}$ is a symmetric matrix, then the values of x, y, z are

(a) 2, -1,3 (b)2,-2,1 (c) 1,-2,2 (d) none of these.

(xi) $\frac{d}{dx}(a^x)$ = ?

(a) a^x (b) log a (c) a^x log a (d) 0

TURN OVER

Q2. (a) Attempt any two (4 marks each) **[8]**

(i) Find the N^{th} derivative of $y=\frac{x}{(x+2)(x-2)}$

(ii) State the Lagrange's Mean Value theorem. Use it to verify for $f(x) = \sin^{-1}x$ in [0,1]

(iii) Using Taylor's series, expand sin x in ascending powers of $(x - \frac{\pi}{2})$.

(b) Attempt any one (3 marks) **[3]**

(i) If $y=x^n \log x$, Show that: $y_{n+1} = \frac{n!}{x}$ using Leibnitz's theorem.

(ii) Verify Rolle's theorem for the function $f(x)=x^2-3x+2$ in [1,2]

Q3. (a) Attempt any two (4 marks each) **[8]**

(i) Evaluate: $\int e^{3x} sin4x \ dx.$

(ii) Prove that: $\int_0^{\frac{\pi}{2}} \frac{cosx}{sinx+cosx} dx = \frac{\pi}{4}$

(iii) The loop of the curve $9y^2 = x(x-2)^2$ rotated about x – axis. Find the volume of the solid formed.

(b) Attempt any one (3 marks) **[3]**

(i) Evaluate: $I = \int \frac{cosx}{4sin^2x-25} dx.$

(ii) By using the properties of Definite Integral, Prove $\int_0^{\pi/2} (\frac{cosx}{sinx+cosx}) \, dx = \frac{\pi}{4}$

Q4. (a) Attempt any two (4 marks each) **[8]**

(i) Evaluate: $\begin{vmatrix} 1+x & 2 & 3 & 4 \\ 1 & 2+x & 3 & 4 \\ 1 & 2 & 3+x & 4 \\ 1 & 2 & 3 & 4+x \end{vmatrix} = 0$

(ii) Solve by Cramer's rule:- x + y = 3; y + z = 5; x + z = 4

(iii) By using the Adjoint method, find the inverse of the matrix $A=\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

(b) Attempt any one (3 marks) **[3]**

(i) Find the Rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

(ii) If $A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ then find 2A + B.

Q5. (a) Attempt any two (4 marks each) **[8]**

(i) Find the particular solution of the differential equation $\frac{dy}{dx} - x = xy^2$, if y = 1, when x = 0.

(ii) Solve the differential equation: $(x + y)\frac{dy}{dx} = y$

(iii) Find the particular solution of: $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, when x=0, y=1 and $\frac{dy}{dx} = 0$.

(b) Attempt any one (3 marks) **[3]**

(i) Form the differential equation for $y = ae^x + be^{-x}$, where a,b are arbitrary constants.

(ii) Solve the D.E by using substitution: $(x+y)^2\frac{dy}{dx} = 1$

Q6. (a) Attempt any two (4 marks each) **[8]**

(i) The following data gives the no. of defectives articles by workers in a factory in a month. Find the arithmetic mean.

No. of defective articles	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of workers	5	8	10	12	5

(ii) For the following grouped data, find the modal value:

I.Q. Group	10-30	30-50	50-70-	70-90	90-110	110-130	130-150
No. of students	5	10	25	30	15	10	5

(iii) Obtain the value of median for the following distribution:

Daily Sales(Rs.)	1400-1600	1600-1800	1800-2000	2000-2200	2200-2400	2400-2600
No. of days	12	30	55	40	35	28

(b) Attempt any one (3 marks) **[3]**

(i) The mean monthly salary paid to 300 employees of a firm is Rs.14,700. The mean monthly salary of 200 male employees is Rs.15,050. Find the mean monthly salary of remaining female employees.

(ii) From the following data, find the missing frequency when mean is 15.38

Size	10	12	14	16	18	20
Frequency	3	7	?	20	8	5

Q7. (a) Attempt any two (4 marks each) **[8]**

(i) Calculate the Q.D and its co-efficient for the following data giving the life of 200 tubes.

Life in hours	600 - 800	800 - 1000	1000 - 1200	1200 -1400	1400 - 1600
No. of tubes	20	60	80	30	10

(ii) Hundred students appeared for two examinations. 60 passed the first, 50 passed the second and
30 passed in both. Find the probability that student selected at random
(a) Passed in at least one examinations.
(b) Failed in both the examinations.

(iii) The life time of a certain kind of pace maker has a mean of 300 days and a standard deviation of 35 days. Assuming that the distribution of life times is normal, find the probability of life time of pace makers is;
(1) more than 370 days. (2) less than 265 days
[Given that area between z=0 and z=2 is 0.4772, Given that area between z=0 and z=1 is 0.3413.]

(b) Attempt any one (3 marks) **[3]**

(i) Find k and hence find the expected value of a random variable x and variance for the probability distribution:-

x	2	3	4	5
P(x)	0.1	k	0.4	0.3

(ii) Calculate the standard deviation for the following data giving the bursting pressure of polythene bags.

Bursting Pressure(in kg.)	5-10	10-15	15-20	20-25	25-30
No. of bags	2	8	25	54	11

Q8. (a) Attempt any two (4 marks each) **[8]**

(i) In a cross-breeding experiment with plants at certain species 240 offspring were classified in 4 classes w.r.t the structure of their leaves as follows:

Class	I	II	III	IV	Total
Frequency	21	127	40	52	240

According to theory of heredity, the probabilities of the four classes should be in the ratio 1:9:3:3. Are these data consistent with theory?(Given that the table value of χ^2 with 3 d.f at 5% l.o.s. is 7.815)

(ii) A random sample of 50 patients suffering from a certain disease was given a serum treatment by which 15 patients were relieved of the disease. Find 99% confidence limits for the percentage of patients cured.

(iii) A manufacturing company purchased three new machines of different makes and wishes to decide whether one of them is faster than others in producing certain output. Five hourly production figures are taken at random from each machine and the results are as follows:-

Hour	Machine wise production		
	A_1	A_2	A_3
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

Using ANOVA determine the mean speeds of 3 machines are significantly different ($F_{0.05,2,12} = 3.89$)

(b) Attempt any one (3 marks) **[3]**

(i) A random sample of size 20 from a normal population gives a sample mean of 42 and a sample standard deviation is 6. Test the hypothesis that the population standard deviation is 9. (Given that: table value of χ^2 with 19 d.f at 5% l.o.s. is 15.507)

(ii) The following data present the yield in quintals of corn on ten subdivisions of equal area of two agriculture plot:

Plot 1:	6.2	5.7	6.5	6.0	6.3	5.8	5.7	6.0	6.0	5.8
Plot 2:	5.6	5.9	5.6	5.7	5.8	5.7	6.0	5.5	5.7	5.5

Test whether two samples taken from two random population have the same variance at 5% l.o.s.
(Given that: the table value F distribution $F_{0.05}(9,9) = 3.1789$.