

(3 Hours)

[Total Marks: 80]

Note: All Questions are compulsory
Use of simple calculator is allowed
Figure at right indicate maximum marks

- Q1. (a) Attempt any 7 [2 marks each]:** **[14]**
- (i) If $y = \frac{1}{3x+2}$ then the n^{th} derivative is;
- a) $\frac{(-1)^n n!}{(3x+2)^{n+1}}$ b) $\frac{(-1)^n n!(3)^n}{(3x+2)^{n+1}}$ c) $\frac{(-1)^{n+1}(n+1)!}{(3x+2)^n}$ d) $\frac{(-1)^{n-1}(n-1)!3^n}{(3x+2)^n}$
- (ii) $\int e^x [f(x) + f'(x)] dx =$
 (a) $e^x f(x) + c$ (b) 0 (c) $e^x \log x + c$ (d) $e^x + c$
- (iii) General solution for the differential equation $(D^3 - 6D^2 + 9D)y = 0$ is:
 (a) $(c_1x + c_2)e^{3x} + c_3$ (b) $c_1e^{3x} + c_2e^{3x} + c_3e^{0x}$ (c) $(c_1x + c_2x)e^{3x} + c_3$ (d) $(c_1x + c_2)e^{3x} + c_3e^{3x}$
- (iv) If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ then the inverse of the A is:
 (a) $\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} -4 & 7 \\ -5 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$
- (v) For a set 50 observations the sum of their squares is 3050, the S.D of the observation is 5.
 Find A.M.
 (a) $\bar{x} = 4$ (b) $\bar{x} = 5$ (c) $\bar{x} = 6$ (d) None of these
- (vi) In a 3 coin trial, the probability of getting at least one Head is
 (a) $1/8$ (b) $3/8$ (c) $5/8$ (d) $7/8$
- (vii) The table value for a Normal distribution $P[Z \geq 2]$ is 0.0228 then $P[-2 \leq Z \leq 2]$ is
 (a) 0.4772 (b) 0.9544 (c) 0.0456 (d) 0.0114
- (viii) For a binomial distribution mean=4 and variance=2.4 then the values of parameters n and p is
 (a) 10 & 0.04 (b) 10 & 0.4 (c) 5 & 0.4 (d) 5 & 0.2
- (ix) If 75% of the items lies above 33.64 and 75% of the items lies below 57, then co-efficient of Quartile deviation is:
 (a) 23.36 (b) 11.68 (c) 0.2573 (d) 0.2537
- (b) Attempt any 1:** **[1]**
- (x) A matrix with all its diagonal elements as one and non-diagonal elements as zero is known as a;
 (a) Zero matrix (b) Unit matrix (c) diagonal matrix (d) none of these
- (xi) If $y = u \cdot v$, where u and v are functions of x, possessing derivative of n^{th} order, then the n^{th} derivative is given by;
 (a) Rolle's Theorem (c) Lagrange's Mean Value Theorem
 (b) Leibnitz's Theorem (d) None of these

TURN OVER

- Q2. (a) Attempt any two (4 marks each) [8]**
- (i) Find the N^{th} derivative of $y = \frac{x}{(x+3)(x-2)}$
- (ii) State Rolle's Mean Value Theorem. Use it to verify for $f(x) = x^2 - 5x + 6$ in $[2, 3]$
- (iii) Using Taylor's series, expand $\sin x$ in ascending powers of $(x - \frac{\pi}{2})$
- (b) Attempt any one (3 marks) [3]**
- (i) If $y = x^3 \log x$, find: y_4 using Leibnitz's theorem.
- (ii) Verify LMVT for the following function: $f(x) = x(x-1)(x-2)$ in $[0, 1/2]$.
- Q3. (a) Attempt any two (4 marks each) [8]**
- (i) Evaluate: $\int e^{5x} \cos 2x \, dx$.
- (ii) Prove that: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} \, dx = \frac{\pi}{4}$
- (iii) Find the volume generated by revolving the arc of the curve $y = \sin x$, between the $x=0$ and $x = \pi$
- (b) Attempt any one (3 marks) [3]**
- (i) Evaluate: $I = \int \frac{e^x}{16 - e^{2x}} \, dx$.
- (ii) The portion of the curve $x^2 = 9y$ between $x=0$ and $x=3$ is revolved about x -axis. Find the volume of solid of revolution.
- Q4. (a) Attempt any two (4 marks each) [8]**
- (i) Find the particular solution of the differential equation $\frac{dy}{dx} - xy^2 = 3y^2$, if $y = 1$, when $x = 0$.
- (ii) Solve the following homogeneous differential equations: $\frac{dy}{dx} = \frac{xy + y^2}{x^2 + xy}$
- (iii) Find the particular solution of: $(D^2 + D - 2)y = 0$, when $x=0, y=1$ and $\frac{dy}{dx} = 0$
- (b) Attempt any one (3 marks) [3]**
- (i) Form the differential equation for $y = ae^x + be^{-x}$, where a, b are arbitrary constants.
- (ii) Solve the differential equation: $(1-x)dy - (1+y)dx = 0$
- Q5. (a) Attempt any two (4 marks each) [8]**
- (i) Find the Rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -3 & 2 & -4 \\ 1 & 2 & -3 & 4 \\ -2 & -1 & -4 & -3 \end{bmatrix}$
- (ii) Find the inverse of the matrix by adjoint method: $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$
- (iii) Solve by Cramer's rule: $x+y+z = 7$; $x+2y+3z=16$; $x+3y+4z=22$
- (b) Attempt any one (3 marks) [3]**
- (i) Evaluate: $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a(b+c) & b(c+a) & c(a+b) \end{vmatrix}$
- (ii) If $A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ then find $2A - 3B + I$.

Q6. (a) Attempt any two (4 marks each) [8]

- (i) The following table gives the platelets count (in lakh/cmm) from the analysis of the blood samples of five different days in pathology laboratory. Find the average platelets count per patient.

Days	1	2	3	4	5
Platelets count	0.50	0.75	1.00	1.43	1.8
No. of patients	75	85	95	100	50

- (ii) Obtain the value of median for the following distribution:

Daily Sales(Rs.)	1400-1600	1600-1800	1800-2000	2000-2200	2200-2400	2400-2600
No. of days	12	30	55	40	35	28

- (iii) Find the missing frequency for the following data given that the mode of the distribution is 44.

Age(in year)	0 – 20	20 –30	-	40 – 50	50 – 60	60– 70	70 – 80	80– 90
No. of persons	10	10		50	29	15	10	10

(b) Attempt any one (3 marks) [3]

- (i) Discuss the Merits and Demerits of Arithmetic Mean.
- (ii) The A.M of 50 observations was found to be 115. It was later noticed that the observation 78 was misread as 87. Find the correct value of the A.M.

Q7. (a) Attempt any two (4 marks each) [8]

- (i) The no. of runs scored by two cricketers A and B in 10 innings of 5 test matches are shown below:
Find which cricketer is more consistent.

A	5	20	90	76	102	90	6	108	20	16
B	40	35	60	62	58	76	42	30	30	20

- (ii) A certain drug is given to two patients. Probability that the patient A will recover is $\frac{2}{3}$ and that of Patient B will recover is $\frac{3}{4}$. Find the probability that
- Both the patients will recover.
 - Both the patients will not recover.
 - Drug is effective.

- (iii) Means and standard deviation are given below for two groups. Find the combined mean and standard deviation of the two groups taken together. They are $n_1=100, \bar{x}_1=40, \sigma_1=5$ and $n_2=200, \bar{x}_2=43, \sigma_2=4$

(b) Attempt any one(3 marks) [3]

- (i) The incidence of occupational disease in an industry is such that the workmen have a 20% chance of suffering from it. What is the probability that out of six workmen, 4 or more will contact the disease?
- (ii) A box contains 7 Aspirin, 5 Analgin and 4 paracetamol tablets. Two tablets are drawn at random from the box. Find the probability that
- both the tablets are Aspirin.
 - one is Analgin and the other is paracetamol.

TURN OVER

Q8. (a) Attempt any two (4 marks each) [8]

(i) Mean weekly sales of chocolate bars in candy stores was 143.5 bars per store. After an ad campaign mean weekly sales in 12 stores for a week increased to 153.7 with S.D. 17.2 bars. Was ad campaign successful?

(Test at $\alpha=0.05$. Use $t_{0.05,11}=1.796$)

(ii) In a sample of 300 tablets manufactured by Pharma company A, 65 tablets were found to be defective and in another sample of 200 tablets manufactured by another Pharma company B, there were 35 defective. Is there any significant difference in the proportion of defective tablets in the sample at 5% l.o.s?

(Given that at 5% significance level table value of $Z_{\alpha/2}$ is 1.96)

(iii) In a preclinical study, animals were treated with two antihypertensive experimental drugs and a control drug with 12 animals randomly assign to three groups four per group. The results(change in blood pressure from baseline) are shown in following table.[Use 5% l.o.s.]

Drug	Drug	Control
1	2	
15	8	-
12	14	16
19	13	20
11	6	22

Use ANOVA technique, given that $F_{0.05}(2,8) = 4.46$.

(b) Attempt any one (3 marks) [3]

(i) From a random sample of size $n=9$ is drawn from normal population gave the following observations:

72, 74, 68, 70, 61, 63, 69, 73 and 71.

To test: $H_0 : \sigma^2 = 36$ Vs $H_1 : \sigma^2 \neq 36$ (Use at 10% l.o.s.)

(Given that table value of χ^2 with 8 d.f at 5% l.o.s. is 2.306)

(ii) In a sample of 8 observations the sum of squared deviations of items for the mean was 94.5. In another sample of 10 observations, the value was found to be 101.7. Test whether the difference in variances is significant at 5% l.o.s?

(Given that the table value of F distribution at (7,9) d.f with 5% l.o.s. is found to be 3.29)