Paper / Subject Code: 65106 / Mathematics

Q. P. Code : 38398

Duration: 3 Hrs **Maximum marks: 70**

Note: **All** Questions are **compulsory**.
Use of **simple calculator** is **allowed**.
Figure at right indicate **maximum marks**.

Q.1 (a) Attempt any 7 [2 marks each] **[14]**

(i) If $\begin{vmatrix} 6 & 2 \\ x+1 & 3 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 1 & 7 \end{vmatrix}$ then the value of x is:
(a) 3 (b) -5/4 (c) 4/5 (d) 5/4

(ii) With respect to Rolle's theorem the value of 'c' corresponding to $f(x)=x^2-4x+3$ is:
(a) 1 (b) 2 (c) 3 (d) 4

(iii) The value of $\int_0^1 (2x + 3x^2 + 4x^3 + 1)dx$ is:
(a) 0 (b) 1 (c) 3 (d) xl4

(iv) If $y=2x^2$, then Δy by taking h=1 is:
(a)2x+1 (b) 4x+2 (c) $2x^2-2x$ (d) $2x^2-1$

(v) If A=$\begin{bmatrix} k & k & 4 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ is a singular matrix, then the value of k is:
(a) 5/4 (b) 5/2 (c)1 5/4 (d) 40/8

(vi) The Nth derivative of f(x)=log(2x+1) is:
(a) $y_n=\frac{1}{2(2x+1)}$ (b) $y_n=\frac{(1)^{n-1}(n-1)!2^n}{(2x+1)^n}$ (c) $y_n=\frac{(1)^n(n)!2^n}{(2x+1)^n}$ (d)$y_n=\frac{(1)^n(n-1)!2^n}{(2x+1)^n}$

(vii) General solution for the differential equation$(D^3-6D^2+9D)y=0$ is:
(a) $(c_1x+c_2)e^{3x}+c_3$ (b) $c_1e^{3x}+c_2 e^{3x}+ c_3 e^{0x}$ (c) $(c_1x+c_2x)e^{3x}+c_3$ (d) $(c_1x+c_2)e^{3x}+c_3e^{3x}$

(viii) The partial derivative of $Z=3x^2+2xy+xy^2$ with respect to x is:
(a) 6x+2y+2xy (b) $6x+2y+2y^2$ (c) $3x+2y+y^2$ (d) $2x+xy+xy^2$

(ix) The inverse of the matrix A=$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ is:
(a)$\frac{1}{14}\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$ (b) $\frac{1}{12}\begin{bmatrix} -3 & 4 \\ -2 & 2 \end{bmatrix}$ (c) $\frac{1}{14}\begin{bmatrix} 2 & -4 \\ -2 & 3 \end{bmatrix}$ (d) $\frac{1}{14}\begin{bmatrix} 2 & 2 \\ -4 & 3 \end{bmatrix}$

(b) Attempt any 1: **[1]**

(x) Which of the following is not a homogeneous differential equation?
(a) f(x,y)=2x-9y (b) $f(x,y)=3x^2-7y^2$ (c) $f(x,y)= x^2+3y^2-1$ (d) a and b

(xi) The value of $\int_{-1}^{1} 3x^3 dx$ is:
(a)15/2 (b) 16/3 (c) 0 (d) 3/2

Q2. (a) Attempt any two (4 marks each) **[8]**

(i) Find the Nth derivative of $y = \frac{x}{x^2-4}$

(ii) State the Lagrange's Mean Value theorem. Use it to verify for $f(x)=x^2-5x+6$ in [2,4]

(iii) Prove that:- $U_{xx} + U_{yy}=0$, where $U=e^x \cos y$

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(b) **Attempt any one (3 marks)**

(i) State Roll's Mean Value Theorem. Use it to verify for $f(x) = x^2 - 5x + 6$ in [2, 3]

(ii) Find the N^{th} derivative of $y = \frac{x+1}{x^2-4}$

Q3. (a) Attempt any two (4 marks each) **[8]**

(i) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} sin^n x\, dx$, hence evaluate $\int_0^{\frac{\pi}{2}} sin^7 x\, dx$.

(ii) Find the volume generated by revolving the arc of the curve y = sinx, between the lines x=0 and x= π

(iii) Evaluate: $\int e^x cosx\ dx$

(b) **Attempt any one (3 marks)** **[3]**

(i) The loop of the curve $y^2 = x(x-1)^2$ rotates about x-axis. Find the volume of the solid formed.

(ii) By using the properties of Definite Integral Evaluate $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+cotx} dx$

Q4. (a) Attempt any two (4 marks each) **[8]**

(i) By using the Adjoint method, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

(ii) Find the Eigen values and one of the Eigen vectors of the matrix:
$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

(iii) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(b) Attempt any one(3 marks) **[3]**

(i) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

(iii) Verify Cayley Hamiltom theorem for the matrix:-$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q5. (a) Attempt any two (4 marks each) **[8]**

(i) solve $(x^3 + y^3)dy = x^2y\,dx$

(ii) Form the differential equation for y = A cos(logx) + B sin(logx)

(iii) Find the particular solution of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$, when x = 0, y = 1 and $\frac{dy}{dx}=0$

(b) Attempt any one(3 marks) **[3]**

(i) Form the Differential Equation of $x^2+y^2=a^2$, where a is an arbitrary constant.

(ii) Solve the differential equation: $x\frac{dy}{dx} = y-x$

Q6. (a) Attempt any two (4 marks each) **[8]**

(i) If f(1) =1, f(4) = -1, f(6) =1, evaluate f(2) using Lagrange's interpolation formula.

(ii) Find the approximate value of $\int_0^8 (1+x^2)dx$, using Trapezoidal rule (take n = 8)

(iii) Estimate the missing term by using E and Δ from the following:

x:	1	2	3	4	5
y:	4	8	--	22	32

(b) Attempt any one(3 marks) **[3]**

(i) Given:

x	1	2	4
f(x)	2	6	24

Estimate f(3) by constructing difference table and making a suitable assumption.

(ii) Evaluate: $\left(\frac{\Delta^2}{E}\right)x^4$, (h=1)
